Higher Order Modes in Suspended Substrate Microstrip Lines

Ioannis P. Polichronakis and Stamatis S. Kouris

Abstract— **The characteristics of both types of higher order modes in suspended substrate microstrip lines are evaluated using Spectral Domain Analysis. Also, the cutoff frequencies of the first two higher order modes are computed and the operation bandwidth is evaluated. The obtained results are compared with some estimated values given by other authors. Finally, design curves are reported for practical suspended substrate microstrip line structures.**

I. Introduction

The behavior of the dominant mode of suspended substrate microstrip lines has been studied by various authors [1]–[4]. However, it is evident that, in practical applications, we need to know:

1) the behavior of the higher order modes (even and odd) in the line;

2) the structural parameters of the line (i.e., strip width, relative dielectric constant of the dielectric substrate, etc.), which will ensure the best operation of the line under the dominant mode; and

3) the parameters of the line which will produce a certain characteristic impedance and a certain line wavelength.

There is not much information available about the higher order modes in an SSL, and therefore about the bandwidth of operation of the dominant mode. Some results from an approximate analysis have been published by Gardiol [5]. Still, a complete study of the line concerning the higher order modes of both types (even and odd) and their cutoff frequencies seems not to have been presented yet.

The purpose of this paper is to describe the behavior of the higher order modes in an SSL using the Spectral Domain Approach (SDA). The results of this analysis regarding the behavior of the line, with respect to its structural parameters and the determined cutoff frequencies of the first two higher order modes, may be used to estimate the best operational bandwidth of the dominant mode. Some design curves are also reported that can be used to evaluate the structural parameters of an SSL when its characteristic impedance is 50 Ω.

II. Analysis

The line is assumed to be lossless and the strip thickness negligible. The cross section of an SSL is shown in Fig. 1. The propagation constant β of the dominant and higher order modes can be obtained using the Spectral Domain Approach [4]. The set of basis functions used in this analysis is the set proposed by Jansen [7], because these basis functions provide fast and accurate results, as it has been shown elsewhere [6].

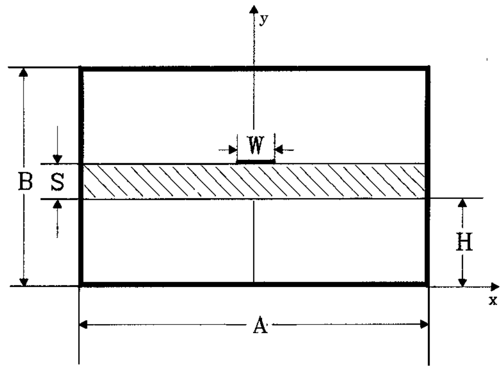

Fig. 1. Cross section of an SSL line. The outer dimensions of the waveguide shielding are $A \times B$, W is the strip width, S is the substrate thickness, and H is the substrate distance from the bottom wall of the waveguide.

For the Even Modes:

$$J_{Zo} = 1/\left[1-\omega^2\right]^{1/2}$$
$$J_{Zn} = \left[\cos(n\pi\omega) - J_o(n\pi)\right]\left[1-\omega^2\right]^{-1/2}$$
$$J_{Xo} = 0$$
$$J_{Xn} = \int_0^{\omega} \left[\cos(n\pi v) - J_o(n\pi)\right]\left[1-v^2\right]^{-1/2} dv$$
$$n = 1, 2, \cdots, \infty.$$

For the Odd Modes:

$$J_{Xm} = \cos(m\pi\omega)\left[1-\omega^2\right]^{1/2}$$
$$J_{Zm} = \frac{d}{d\omega}\left[\cos(m\pi\omega)\left[1-\omega^2\right]^{1/2}\right] \qquad m = 0, 1, \cdots, \infty$$

where J_o is the zero-order Bessel function of the first kind, $\omega = 2x/W$, and W is the strip width (Fig. 1). In the present analysis for the even modes, it is given that $n = 0$ and $n = 1$, and for the odd modes $m = 0$. Moreover, the characteristic impedance can be computed using the well-known power-current definition: $Z_o = 2 \times P_{\text{avg}}/I^2$, where $I = \int_{-w/2}^{w/2} J_Z(x, L)\, dx$, and $L = H + S$ is the distance of the strip from the bottom wall of the shielding waveguide (Fig. 1).

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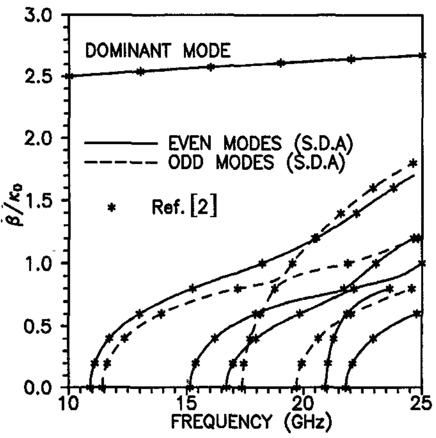


Fig. 2. Higher order modes in a microstrip line using SDA. Solid lines represent even modes, while dotted lines represent odd modes. The points represent the results obtained by Yamashita [2].

III. Numerical Results and Discussion

A. Higher Order Modes

A computer program has been developed to calculate the propagation constant β of the dominant and higher order modes in the line. The lack of published results concerning the higher order modes in an SSL has forced us to check the accuracy of the analysis regarding the basis functions and the number of spectral terms to be assumed. This has been done by computing [6] the values of the propagation constant β of the dominant and higher order modes of a microstrip line, that is, for $H = 0$. The microstrip is placed in a waveguide with outer dimensions $A = 12.7$ mm and $B = 12.7$ mm, strip width 0.635 mm, on a substrate of thickness 1.27 mm and dielectric constant $\epsilon_r = 8.875$. The results obtained using SDA are shown in Fig. 2, together with those obtained, for the same line, by Yamashita *et al.* [2], using the method of nonuniform discretization of integral equations. It is evident from this figure that a close agreement exists between the results of the two methods.

Then, the dispersion characteristics for even and odd modes are computed for different dielectric substrates: 1) $\epsilon_r = 3.78$, 2) $\epsilon_r = 9.8$, and 3) $\epsilon_r = 17$, and a ratio $S/B = 0.1$. These results are illustrated in Fig. 3, and refer to an SSL placed on a WR-28 waveguide which is usually used in the 26.5–40 GHz band. On the other hand, Fig. 4 shows the higher order modes of this line when the substrate thickness is 1) $S/B = 0.2$ and 2) $S/B = 0.3$, and the relative dielectric constant is 3.78.

Figs. 3 and 4 clearly show that for higher values of the relative dielectric constant of the substrate, the line becomes more dispersive. It can also be seen that the "cutoff frequency" of the first even mode is practically independent from the relative dielectric constant of the substrate, whereas the "cutoff frequency" of the first odd mode is highly dependent on ϵ_r. Indeed, when the relative dielectric constant of the substrate is increasing, this frequency is drastically decreasing, so that when ϵ_r is greater than about $\epsilon_r = 17$, the first higher order mode becomes an odd one [Fig. 3(c)].

Furthermore, we may note from Fig. 4 that for substrates with low dielectric constants, a change in their thickness affects all the higher order modes except the first even. Therefore, an increase in the thickness of the substrate may result in a decrease in the cutoff frequency of the first odd mode; therefore, for thick substrates even with moderate relative dielectric constants, the first higher order mode could be an odd one.

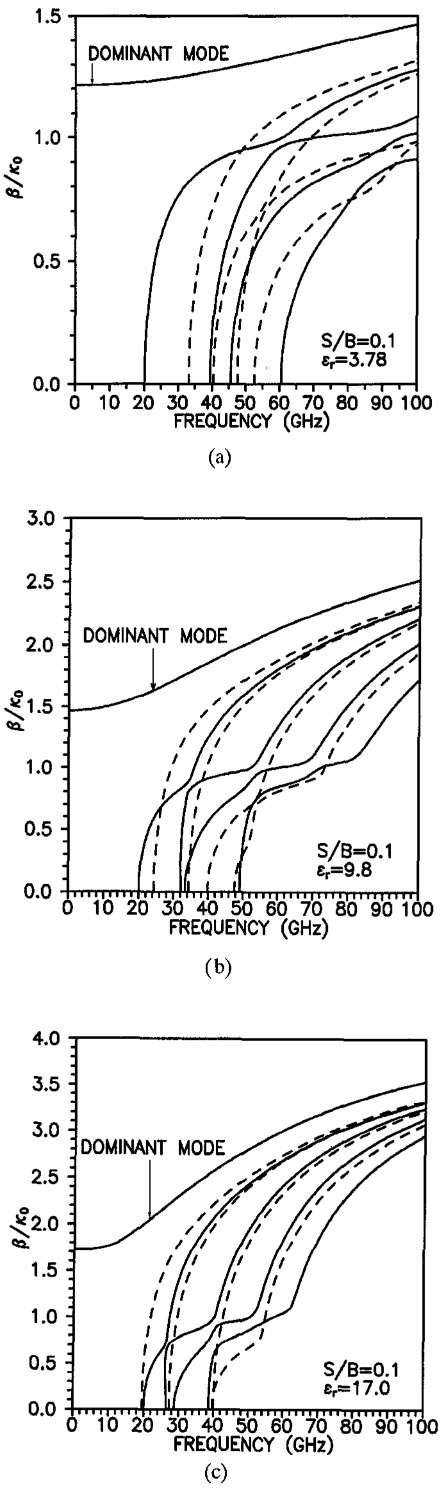
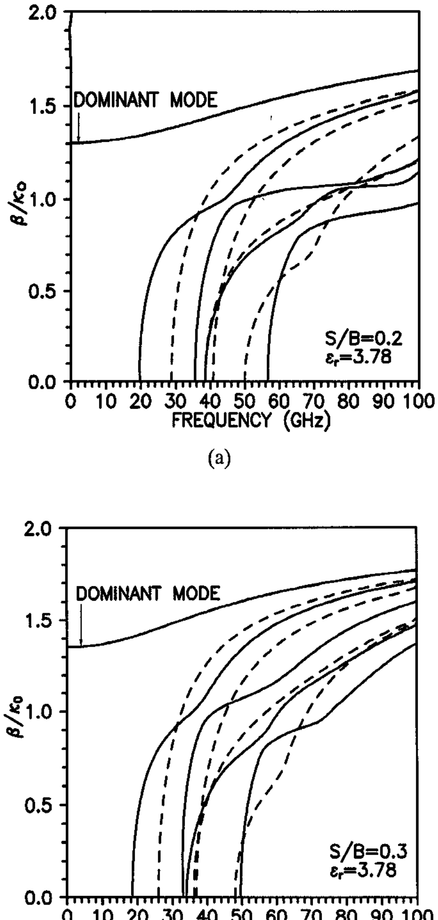


Fig. 3. Higher order modes in an SSL for different relative dielectric constants of the substrate: (a) $\epsilon_r = 3.78$, (b) $\epsilon_r = 9.8$, (c) $\epsilon_r = 17.0$. (WR-28 shielding, $W = 0.1A$, $S = 0.1B$, $H = (B - S)/2$.)



(a)

(b)

Fig. 4. Higher order modes in an SSL for different dielectric substrate thicknesses: (a) $S = 0.2B$, (b) $S = 0.3B$. (WR-28 shielding, $W = 0.1A$, $S = 0.1B$, $H = (B - S)/2$, $\epsilon_r = 3.78$.)

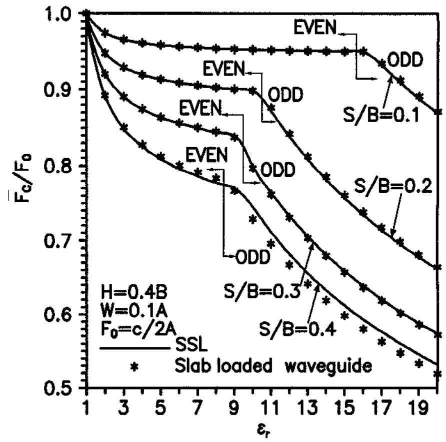


Fig. 5. Cutoff frequency (F_c) of the first higher order mode in SSL for different relative dielectric constants of the substrate and for different substrate thicknesses. F_o is the cutoff frequency of the TE_{10} waveguide mode. Asterisks represent the cutoff frequency in a similar dielectric slab loaded waveguide. (WR-28 shielding, $W = 0.1A$, $H = 0.4B$.)

The above variations in the cutoff frequencies of the modes are due to the variations in the electrical dimensions due to the changes in the field distributions in the dielectric. For instance, the small variations in the cutoff frequency of the first even mode contrary to the others result from the fact that the electrical dimensions of the line in this case are little affected by variations in ϵ_r and/or S.

B. Operation Bandwidth

From the above-mentioned results, it is evident that the operation bandwidth of the line, limited by the cutoff frequency of the first higher order mode, depends on its structural parameters. The first higher order mode could be either an even or an odd one. Therefore, the cutoff frequency of both the first even and the first odd mode must be computed. Fig. 5 shows the cutoff frequency of the first higher order mode in an SSL as a function of the relative dielectric constant of the substrate for four different thicknesses. The cutoff frequency (F_c) is normalized to the cutoff frequency of an empty waveguide $F_o = c/2A$, where c is the velocity of the light in the vacuum, and A is the wider dimension of the waveguide. It is evident from this figure that for thin substrates ($S/B = 0.1$), the first higher order mode becomes an odd one only for high values of the relative dielectric constant. On the other hand, for thicker substrates, this is true even for lower relative dielectric constants (i.e., $\epsilon_r > 9$). It is evident that an increase in the substrate thickness leads to a linear decrease in the cutoff frequency of the first even mode, and to a nonlinear decrease in the cutoff frequency of the first odd. It can be stated that when substrates with a relative dielectric constant greater than $\epsilon_r = 10$ are used, care should be given to their thickness. Indeed, Fig. 5 points out the fact that for a value of $\epsilon_r = 10$ and even for a substrate thickness $S/B = 0.2$, the first higher order mode is an odd, and the operation bandwidth of the line becomes narrower.

We may conclude, therefore, that an increase in the relative dielectric constant and/or the thickness of the substrate leads generally to a decrease in the operation bandwidth of the line. Of course, this is especially valid when the first higher order mode is an odd.

In Fig. 6 we report the results regarding the variation of the cutoff frequency of the first higher order mode against ϵ_r for different distances of the substrate from the bottom wall of the waveguide shielding. It can be seen that by moving the substrate toward the bottom wall, the first higher order mode becomes an even for much greater values of ϵ_r than those when the substrate is in the middle of the shielding. For instance, for a line with a substrate of relative dielectric constant $\epsilon_r >= 17$ placed at a height $H = 0.4B$ from the bottom wall of the waveguide shielding, the first higher order mode is an odd. Now if the height is decreased from $H = 0.4B$ to $H = 0.2B$, the first higher order mode is an odd only when ϵ_r is greater than 20. Therefore, placing a substrate of a given thickness, closer to the bottom wall of the waveguide shielding, the first higher order mode will be an even if ϵ_r is not to high, i.e., $\epsilon_r < 16$.

Moreover, we can see from the same Fig. 6 that by moving the dielectric substrate toward the bottom wall of the shielding, the variation of the cutoff frequency is small when the first higher order mode is an even, whereas it is much greater when this mode is an odd.

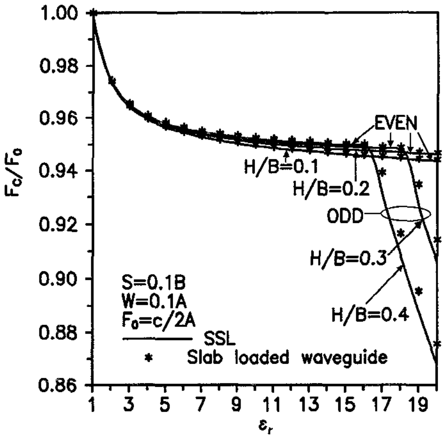


Fig. 6. Cutoff frequency (F_c) of the first higher order mode in an SSL for different dielectric constants of the substate and for different substrate distances from the bottom wall of the waveguide shielding. Asterisks represent the same frequencies as in Fig. 5. (WR-28 shielding, $S = 0.1B, W = 0.1A$.)

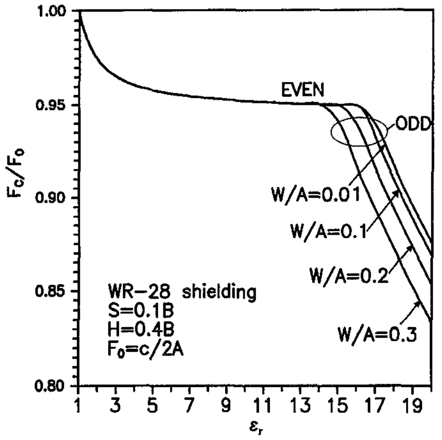


Fig. 7. Cutoff frequency (F_c) of the first higher order mode in an SSL for different dielectric constants of the substrate and different strip widths. Asterisks represents the same frequencies as in Fig. 5. (WR-28 shielding, $S = 0.1B, W = 0.1A, H = 0.4B$.)

Finally, the effect of the strip width on the operation bandwidth of the line is illustrated in Fig. 7. It can be seen that the operation bandwidth is not strongly affected except for substrates with a very high relative dielectric constant. Indeed, the change in the strip width affects only the first odd mode. This may be explained by the fact that this mode has a magnetic wall along the x-axis (maximum in the electric field). Therefore, there is a capacitive coupling between the strip ends and the vertical wall of the shielding; that makes the structure look electrically larger in the y direction than it is. Since the first odd mode is a distorted TE_{01} waveguide mode having a cutoff frequency $f_0 = c/2B$, the actual cutoff frequency F_c should be smaller. Thus, for large ratios of W/A and substrates of high relative dielectric constants, the first higher order mode could be an odd one.

In Figs. 5 and 6, we have also reported (noted by asterisks) the estimated values of the cutoff frequency of the first higher order mode for a similar dielectric slab loaded waveguide, using the approximation method proposed by Gardiol [5]. It is evident that this approximation is valid mainly in the case of the first higher order mode being an even one. However, when the first higher order mode is an odd, this approximation technique generally leads to an overestimation of the cutoff frequency, which is due to the fact that it neglects the effect of the strip on the cutoff frequency of this mode.

It should be noted that the behavior of the line is indenpendent of the sizes of the waveguide. It only depends on the ratio A/B. Indeed, when other waveguide shieldings (e.g., WR-19, WR-22) are used, the results of Figs. 5–7 are identical. This means that the results of Fig. 5–7 can be used for every line with waveguide shieldings that have an aspect ratio $A/B = 2$.

The above-reported results lead us to the conclusion that the operation bandwidth of the line is upward limited by the cutoff frequency of the dominant mode of the chosen waveguide shielding $F_o = 2c/A$. Furthermore, the main factors affecting the operation bandwidth are the thickness and the relative dielectric constant of substrate. The effects of the strip width and the substrate height from the bottom wall of the waveguide shielding are important only when the first higher order mode becomes an odd one. It is therefore concluded that the maximum operation bandwidth of the dominant mode for a given waveguide shielding can be achieved by using a narrow strip printed on thin substrate with low relative dielectric constant, and placing the substrate near the bottom wall of the waveguide shielding. This bandwidth is only 4–5% lower than the cutoff frequency of the TE_{10} mode of the shielding.

IV. Design Curves

In general, it is rather difficult to present design curves for a line like the one treated here since there exist several independent variable parameters which describe the structure. The difficulties may be overcome to some extent if some practical considerations are taken into account, such as those, for example, adopted by Knorr and Shayda for a fin-line [8]. Indeed, it is convenient to produce design curves for shieldings which have dimensions similar to those of the standard rectangular waveguides used in the millimeter-wave band, that is, shieldings like WR-28, WR-19, WR-12, which cover the frequency up to 100 GHz and have an aspect ratio $A/B = 2$. In most of the practical cases, the strip is printed on a substrate of RT/duroid which has a relative dielectric constant $\epsilon_r = 2.22$ or is on a substrate of fused quartz with $\epsilon_r = 3.78$. The strip is centered in the guide.

The main problem that arises in the design of an SSL is determining the adequate line's structural parameters, which will give a certain value of the characteristic impedance and the line wavelength. Usually, a 50 Ω characteristic impedance is adopted, and the design curves reported in Figs. 8 and 9 are referred to such a line when the substrate is 1) RT/duroid with $\epsilon_r = 2.22$ (Fig. 8), and 2) fused quartz with $\epsilon_r = 3.78$ (Fig. 9). From these curves, the strip width and the substrate thickness may be evaluated. Moreover, the guide wavelength of the line can also be determined. For instance, the design of a 50 Ω line on an RT/duroid substrate at the frequency of 30 GHz can be done with the aid of Fig. 8: choosing a shielding with outer dimensions $A = 10$ mm and $B = 5$ mm, that is, $A/\lambda_o = 1$, and a ratio $S/B = 0.2$, the line must have [Fig. 8(a)] a W/A

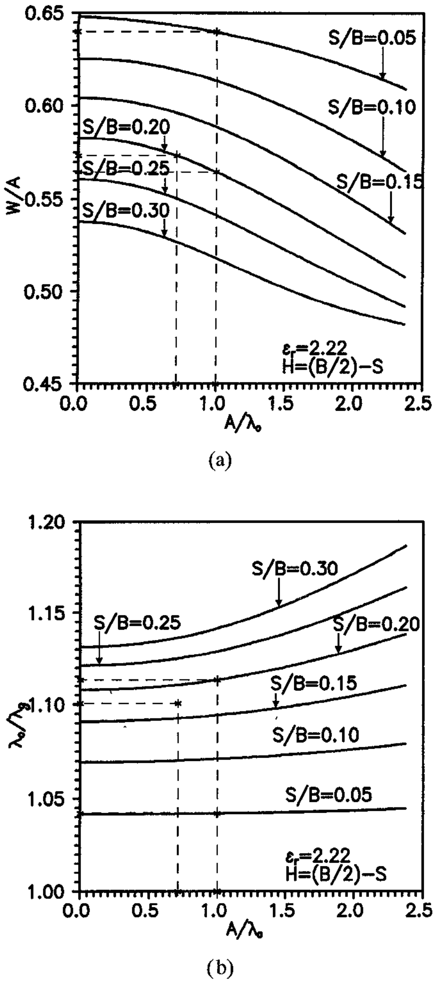


Fig. 8. (a) Strip width and substrate thickness of a 50 Ω SSL on an RT/duroid substrate. The substrate is centered in the waveguide shielding. (b) The corresponding guide wavelength of the line.

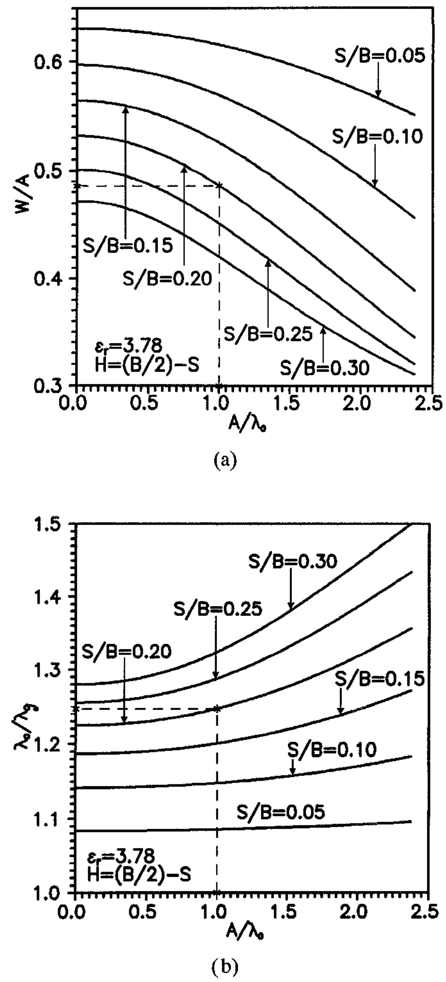


Fig. 9. (a) Strip width and substrate thickness of a 50 Ω SSL on a fused quartz substrate. The substrate is centered in the waveguide shielding. (b) The corresponding guide wavelength of the line.

ratio of 0.5652. If the S/B ratio is 0.05, then the W/A ratio is 0.641. From Fig. 8(b), the corresponding guide-wavelength is determined. Indeed, the corresponding ratio λ_o/λ_g becomes 1.1125 in the first case and 1.042 in the second case.

If smaller waveguide shielding is chosen, e.g., WR-28 with outer dimensions $A = 7.112$ mm and $B = 3.556$ mm, then, at the same frequency of 30 GHz, the ratio A/λ_o becomes 0.7112. In that case, for the same S/B ratio (that is, $S/B = 0.2$), the resulting W/A ratio is 0.5725 and the corresponding guide wavelength is smaller than in the case of the large shielding ($\lambda_o/\lambda_g = 1.114$).

We may observe (Figs. 8 and 9) that in order to design a line with a characteristic impedance of 50 Ω, a large strip width is generally needed. To achieve the same characteristic impedance using a smaller strip width, a substrate with higher relative dielectric constant could be used. In fact, using fused quartz ($\epsilon_r = 3.78$) instead of RT/duroid ($\epsilon_r = 2.22$) for the same substrate thickness, a smaller strip width is necessary. For example, at the frequency of 30 GHz and with a shielding of outer dimensions $A = 10$ mm and $B = 5$ mm, a line of a 50 Ω characteristic impedance build on a substrate of thickness $S = 0.2 \times B$ must have a strip width $W = 0.5652 \times A$ on an RT/duroid and $W = 0.49 \times A$ on a fused quartz substrate. However, in the latter case, the guide wavelength decreases, making the circuit dimensions even smaller.

V. Conclusions

The behavior of the even and odd higher order modes, in a suspended substrate microstrip line, has been studied for frequencies up to the millimeter-wave region as a function of the various parameters of the line. It is concluded that the main factors affecting the cutoff frequencies of the first even and the first odd mode of the line, and thus the operation bandwidth, are the relative dielectric constant and the thickness of the substrate. The strip width affects only the cutoff frequency of the first odd mode. Moreover, the estimation of the bandwidth from the cutoff characteristics of a similar dielectric loaded waveguide is valid when the first higher order mode is an even one. On the contrary, this approximation technique overestimates the cutoff frequency of the first odd mode, unless the strip width is small.

Finally, the reported design curves could be very useful in practical applications. Regarding these curves, it should be noted that an increase of the substrate thickness and/or the relative dielectric constant of the substrate leads of course to a smaller strip width, but at the same time it results in a smaller operation bandwidth under the dominant mode, as is shown in Figs. 5 and 6. Therefore, an optimization technique should be used in order to obtain the proper values of the structural parameters of the line when these design curves are used.

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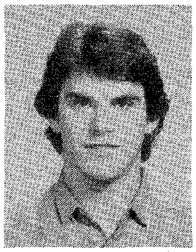

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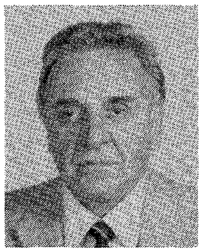

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